Asset and Equity Volatility in Business Valuations

Background, Estimation & Application

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Introduction

- Equity or asset volatility is a key input in a number of calculations used to derive valuations for private companies, complex securities and discounts
- Given the increased use of option-based models and derivative securities, the use of volatility has increased in recent years
- Volatility is generally an unobservable input that must be estimated from public equity markets
- Adjustments to observed volatilities in public equity markets are becoming more common

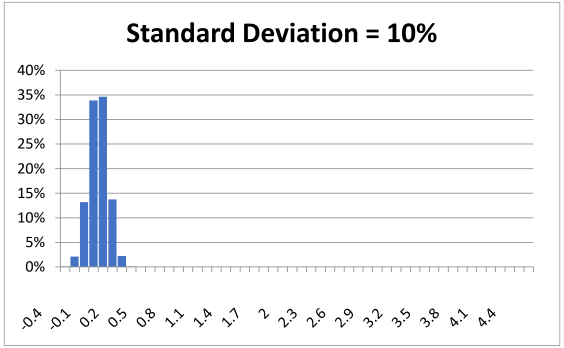
Volatility is a Measure of Risk

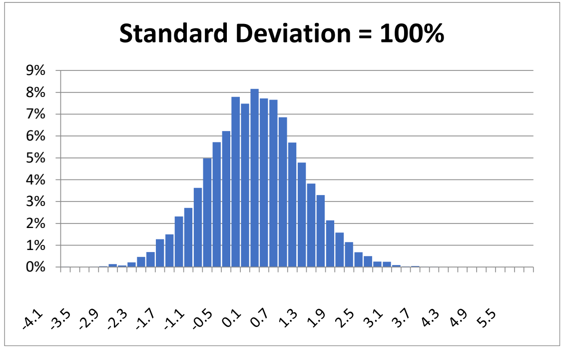
- There is no universally agreed-upon definition of risk, however, the main measures of risk used for financial instruments are **variance** and **standard deviation**
- Risk means both potential upside (gain) and downside (loss)
- Variance, and its square root the standard deviation (typically designated as σ or sigma), are the standard statistical measures of the spread of a sample
- The variance of a single asset can be described by the following formula:

$$\text{Variance} = \sigma^2 = \frac{\sum_{i=1}^{N} (R_{it} - \overline{R}_i)^2}{N - 1}$$

The Dispersion of Equity Returns

- Equity returns are not normally distributed, but it is a useful approximation
- A stock with a high standard deviation will have a high dispersion of returns, in comparison with a stock with a low standard deviation
- A riskless asset would have a standard deviation of zero





Volatility vs. Beta

- Total risk for a stock is measured by volatility
  - Volatility is normally defined as the standard deviation of a stock over a period of time
- Systematic risk for a stock is measured by beta
  - Beta is a measurement of additional risk of adding one stock to a diversified portfolio, or the expected change in the excess return of the security for a 1% change in return of the market portfolio, under the Capital Asset Pricing Model
  - Beta measures the responsiveness of a security to movements in the market portfolio (i.e., systematic risk)
  - Beta is defined as the Covariance of a stock with the equity market, divided by the standard deviation of the equity market

$$\beta_i = \frac{Cov(R_{i,} R_M)}{\sigma^2(R_M)}$$

Volatility Use Cases in Business Valuations

- OPM Backsolve and contingent waterfall analysis
- DLOM option models
- Warrant valuation
- Convertible bond valuation
- Contingent consideration valuation
- Real options analysis

Equity Volatility Estimation

- Methods which can be used to estimate equity volatility in a private company include:
  - Historical (realized) share volatility for guideline public companies
  - Implied share volatility for guideline public companies
  - Actual volatilities used by guideline public companies
  - Volatility of financial metrics (Revenues, EBITDA, EBIT, Net Income) for subject companies
  - Financial projections with Monte Carlo simulation of earnings

Historical (Realized) Volatility Estimation

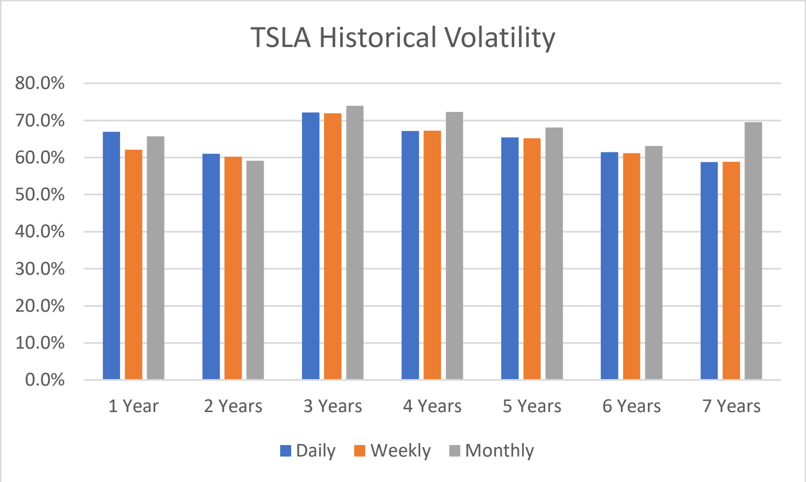
- For business valuation purposes generally done based on daily, weekly or monthly stock price changes (close-to-close)
- Volatility is normally calculated based on the natural log (LN in Excel) price change
  - If using daily prices, annual volatility is calculated based on 252 trading days per year
  - If using weekly prices, annual volatility is calculated based on 52 trading weeks per year
  - If using monthly prices, annual volatility is calculated based on 12 trading months per year
- We can also consider smoothing of the historical volatility using EWMA (exponentially weighted moving average) or GARCH methods, but this is likely not acceptable for ASC reporting
- For (options) trading purposes, realized volatility is sometimes calculated based also on intraday prices (e.g., open, close, high and low)

Tesla (TSLA) Historical Volatility Estimation

- Download weekly closing stock prices for 5 years or more
- Calculate natural log price change from week to week (LN($price_N$/$price_{N-1}$)
- For one-year historical volatility first calculate the Standard Deviation for the 52 most recent weeks = 0.0861 for calendar year 2022
- Then multiply the Standard Deviation with the Square Root of 52 (7.2111), for 52 measurements
- The one-year historical volatility estimate is 0.0861 x 7.2111 = 62.1%
- Two-year historical volatility calculation will be based on 104 measurements and so on
- Historical volatility calculation for periods of up to 5-10 years is common, but some companies may have shorter trading histories, using more than 10 years is discouraged

Historical Volatility Estimates – Daily/Weekly/Monthly Data

- Using daily or weekly trading data generally produces very similar results
- Monthly trading data can be less reliable
- Suggested that monthly calculation should not be used with less than 3 years of data



Implied Equity Volatility

- By looking at trading prices for options on publicly traded shares we can back-solve, using the Black-Scholes-Merton option model, for the volatility that is **implied** by the price, term, and interest rate applicable to an option
- The implied volatility is a **forward-looking** indication of the market's expectation of the stock volatility
- To calculate implied volatility for a past valuation you will need historical options chain data, which is generally only available from paid services
- Averaging of the call and put options implied volatility is suggested
- Options contract data is typically available for up to 1,080 days (~3 years), but further out contracts may have small volumes and therefore less reliable
- Getting already calculated implied volatilities for past dates from a service provider, such as Bloomberg or iVolatility, is often more efficient

Implied Equity Volatility (cont'd)

- While the forward-looking aspect of implied equity volatilities is attractive, there are certain issues to consider:
  - Not all stocks have exchange traded options and trading volumes may be small for some stocks
  - Implied volatilities will typically differ between strike prices and maturities, meaning that the selection process will influence outcome
  - The Black-Scholes-Merton model is not a precise predictor of exchange traded options prices and therefore implied volatilities based on it are only approximations

Implied Equity Volatility Calculation

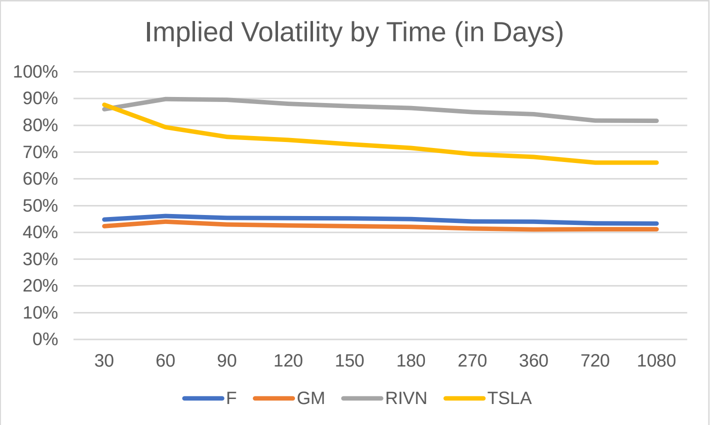
- Example Calculation for TSLA on 6/2/23:
  - Closing price for 6/21/24 call option at $15.00 was $4.50
  - Using Excel Goal-Seek function we can solve for the volatility that results in a call option value of exactly $4.50, which is 76.1%

Black-Scholes Option Pricing Model

Current Stock Price	$	14.48
Exercise Price	$	15.00
Risk-Free Interest Rate		5.22%
Expected Life of Option (Years)		1.0541
Volatility		76.1%
Dividend Yield		0.0%
Call Option Value	**$**	**4.5000**

Implied Equity Volatility – iVolatility Example

Stock	F	GM	RIVN	TSLA
30d iv call	44.6%	42.2%	86.4%	87.9%
30d iv put	45.0%	42.4%	85.7%	87.5%
30d iv mean	44.8%	42.3%	86.0%	87.7%
60d iv call	45.9%	44.1%	89.8%	79.5%
60d iv put	46.4%	43.9%	89.7%	79.1%
60d iv mean	46.1%	44.0%	89.8%	79.3%
90d iv call	45.5%	43.0%	89.4%	75.9%
90d iv put	45.3%	42.8%	89.7%	75.5%
90d iv mean	45.4%	42.9%	89.6%	75.7%
120d iv call	45.4%	42.7%	88.0%	74.7%
120d iv put	45.1%	42.4%	88.2%	74.3%
120d iv mean	45.3%	42.5%	88.1%	74.5%
150d iv call	45.4%	42.5%	87.1%	73.2%
150d iv put	45.0%	42.1%	87.3%	72.7%
150d iv mean	45.2%	42.3%	87.2%	72.9%
180d iv call	45.2%	42.3%	86.4%	71.0%
180d iv put	44.8%	41.8%	86.6%	72.0%
180d iv mean	45.0%	42.1%	86.5%	71.5%
270d iv call	44.5%	41.8%	84.7%	69.6%
270d iv put	43.7%	41.1%	85.1%	69.0%
270d iv mean	44.1%	41.4%	84.9%	69.3%
360d iv call	44.6%	41.5%	83.9%	68.6%
360d iv put	43.4%	40.7%	84.4%	67.8%
360d iv mean	44.0%	41.1%	84.1%	68.2%
720d iv call	44.8%	42.2%	81.9%	67.6%
720d iv put	41.9%	40.1%	81.8%	64.6%
720d iv mean	43.3%	41.2%	81.8%	66.1%
1080d iv call	44.8%	42.3%	81.8%	67.7%
1080d iv put	41.8%	40.0%	81.6%	64.4%
1080d iv mean	43.3%	41.2%	81.7%	66.1%



Actual Volatility From Guideline Public Companies

- SEC reporting companies will often disclose the volatility used (or a range) for the calculation of options expense (or for other equity incentives) in 10-K filings
- This volatility can sometimes be materially different from historical and current implied volatility for the guideline public company
- Disclosure is sometimes also available of volatility used for warrant instruments and may be significantly different from the volatility used for common share options

Earnings Volatility

- Historical earnings or revenue volatility for a subject company is conceptually a good proxy for its equity volatility, as measured by standard deviation
- However, when based on annual financial data we will rarely have sufficient data points for providing a reliable estimate
- If measuring volatility of revenues, EBITDA or EBIT, this would represent asset volatility and needs to be adjusted for leverage in the subject company
- Risk-adjusted financial projections using Monte-Carlo simulation can potentially be used to estimate earnings volatility, as this provides many data points

Considerations for Selection of Volatility Estimate

- Historical volatilities will often be lower with longer measurement periods used because of mean reversion
- AICPA Cheap Stock Guide and others suggest that the measurement period for historical volatility with should be match the term estimated for a calculation (e.g., time to liquidity in OPM backsolve), however there is no obvious theoretical basis for this
- The guide also suggest that 75th percentile volatility from a group of guideline public companies may be appropriate for a smaller private company, migrating to the median of smaller public companies over time
- Historical volatilities are by definition backward looking and therefore have best predictive ability for large, stable companies
- Implied volatilities are forward looking and in theory a better predictor for expected volatility, but can be influenced by short-term stock trading activity and have some inherent measurement issues

Asset Volatility Calculation

- Equity volatility is influenced by the leverage of a company, therefore we must derive an asset volatility (de-lever) for a public company if it is levered before using it as an indication
- Asset volatility can be calculated using either a static or a dynamic calculation (Merton Model), with the latter being the preferred method
- The static method assumes that debt volatility is zero as a simplification
- The Merton Model treats the company equity as a contingent claim on its assets, i.e., an option

Asset Volatility – Static Method

- Ford (F):
  - Measured equity volatility = 46.8%
  - Equity/MVIC = $43.3 bn / $132.2 bn = 32.9%
  - Asset Volatility = Equity Volatility x Market Value of Equity/MVIC = 46.8% x 32.9% = 15.4%
  - To re-lever the asset volatility the formula is Equity Volatility = Asset Volatility / Equity Weight

Asset Volatility – Merton Model

- Developed in the 1970's by Robert C. Merton as an extension to the Black-Scholes-Merton option pricing model
- Models a company's equity as a call option on its assets
- Asset volatility is calculated as follows:
    - Asset Volatility = Equity Volatility / (Asset Value x (N(d1)/Equity Value))
    - Asset Value = Equity Value + (D x EXP(-Rf x T) x N(d2) / N(d1)
    - Where D = book value of net debt, Rf = risk free rate, T = time to maturity, N(d1) and N(d2) from Black-Scholes-Merton option formula
    - Both formulas need to be solved simultaneously – in Excel this can be done using Solver or Iterative Calculations

Asset Volatility – Merton Model

- If Ford (F) has a Net Debt of $88,799 million, we can calculate the asset volatility as follows using the Merton Model, assuming a debt term of 3 years and risk-free rate of 4.55%:
  - D1 = LN(Asset Value/$88,799)+(4.55%+(0.5 x Asset Volatility^2) x 3.0) / ((3.0^0.5) x Asset Volatility)
  - D2 = D1 – ((3.0^0.5) x Asset Volatility)
  - Asset Value = $119,805
  - Asset Volatility = 18.0%
  - The formulas for Asset Value and Asset Volatility are solved simultaneously

Asset Volatility Calculation	F.US
Market Capitalization	43,443
Net Debt	88,799
MVIC	132,242
Market Cap/MVIC	32.9%
Equity Volatility	46.8%
Asset Value	119,805
Equity Value	43,443
Exercise Price	88,799
Risk-free Rate	4.55%
Term	3.0
D1	1.55130
D2	1.23878
N(D1)	0.93959
N(D2)	0.89229
Asset Volatility (Merton)	18.0%

Volatility Re-Levering

- If the subject company is levered, we can re-lever the calculated asset volatility to an equity volatility using the Merton Model as follows:
  - Equity Volatility = Asset Volatility x (Asset Value x (N(d1)/Equity Value))
  - Equity Value = Asset Value x N(d1) – D x EXP(-Rf x T) x N(d2)
  - Where D = book value of net debt, Rf = risk free rate, T = time to maturity, N(d1) and N(d2) from Black-Scholes-Merton option formula
  - Both formulas need to be solved simultaneously
- For a private subject company, we will need to estimate asset value (MVIC) for this calculation

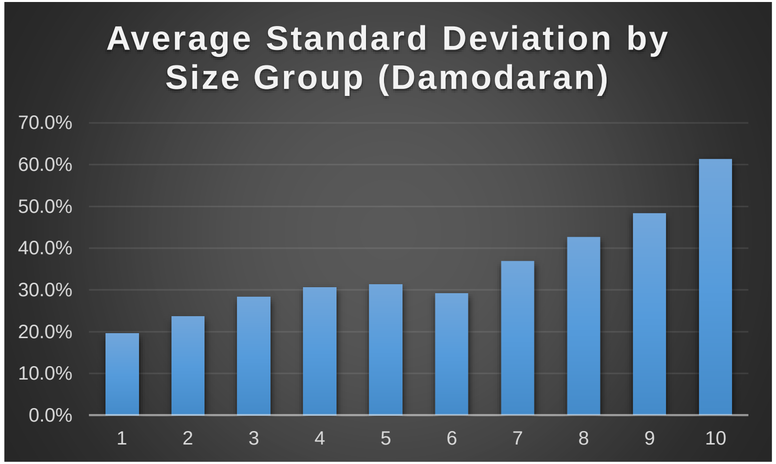
Volatility Re-Levering Example

- Assuming that our subject company, ABC Electric Vehicles, has a Market Value of Invested Capital of $1.5 billion and Net Debt of $350 million, with an estimated asset volatility of 66%, we can calculate its equity volatility at 78.3% using the Merton Model:

Market Value of Invested Capital	1,500.0
Debt	350.0
Market Capitalization	1,218.8
Asset Volatility	66.0%
Equity Volatility	78.3%
Asset Value	1,500.0
Equity Value	1,218.8
Exercise Price	350.0
Risk-free Rate	4.55%
Term	3.0
D1	1.79865
D2	0.65550
N(D1)	0.96396
N(D2)	0.74393

Volatility Size Adjustment

- Volatility is negatively correlated with size in public companies, therefore it is logical to consider a size adjustment when relying on volatility estimates from much larger guideline public companies
- Average volatility by size for public companies is available from Kroll (25 portfolios) and Aswath Damodaran (10 portfolios)
- However, portfolio data is not adjusted for leverage and may have inconsistencies (see chart)
- Also, size adjustment may be inappropriate for some high-risk sectors (e.g., biotech)



Volatility Size Adjustment Example

- Ford (F) has a market capitalization of $44.4 billion, using Damodaran's portfolio #1 its average standard deviation is 19.61%
- ABC Electric Vehicles has an estimated equity value of $1.2 billion, using Damodaran's portfolio #4 average standard deviation is 30.65%
- Volatility multiplier = 30.65% / 19.61% = 1.57
- Size adjusted volatility for Ford is 46.8% x 1.57 = 73.1%

Other Adjustments

- When markets experience unusual turbulence or activity, such as in 2020 with Covid-19, we may consider adjustments or normalization for this – the 3-year historical volatilities in our case study are markedly higher
- Approaches for this can include using a longer historical period, excluding data from the period with unusual activity, or using a mix of historical and implied volatilities

Volatility Analysis Case Study

- We are valuing a promising electric vehicle of company, ABC Electric Vehicles, which has an estimated Market Value of Invested Capital of $1.5 billion, including net debt of $350 million
- As guideline public companies we have selected Ford (F), General Motors (GM), Rivian Automotive (RIVN), and Tesla (TSLA)
- For the volatility analysis we have selected a term of 2 years

Valuation Analysis Case Study (cont'd)

ABC Electric Vehicles

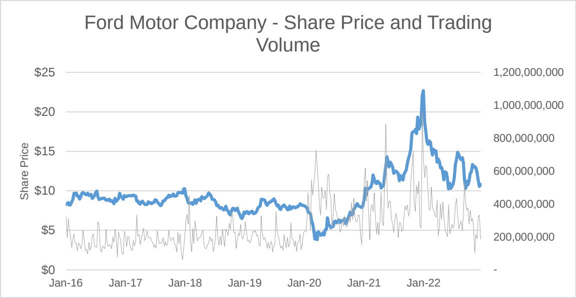
	Company:	Ford Motor Company	General Motors Co.	Rivian Automotive	Tesla	**First Quartile**	**Median**	**Harmonic Mean**	**Mean**	**Third Quartile**	**Low**	**High**
	Symbol:	F.US	GM.US	RIVN.US	TSLA.US							
Look Back Period	1 Year Historical Volatility	51.0%	42.2%	82.9%	62.1%	**48.8%**	**56.5%**	**55.9%**	**59.5%**	**67.3%**	**42.2%**	**82.9%**
	2 Year Historical Volatility	46.8%	40.7%	N/A	60.2%	**43.7%**	**46.8%**	**48.0%**	**49.2%**	**53.5%**	**40.7%**	**60.2%**
	3 Year Historical Volatility	53.7%	51.1%	N/A	72.0%	**52.4%**	**53.7%**	**57.6%**	**58.9%**	**62.8%**	**51.1%**	**72.0%**
	4 Year Historical Volatility	47.9%	46.0%	N/A	67.3%	**46.9%**	**47.9%**	**52.2%**	**53.7%**	**57.6%**	**46.0%**	**67.3%**
	5 Year Historical Volatility	44.5%	43.1%	N/A	65.2%	**43.8%**	**44.5%**	**49.2%**	**50.9%**	**54.8%**	**43.1%**	**65.2%**
	6 Year Historical Volatility	41.1%	40.4%	N/A	61.1%	**40.8%**	**41.1%**	**45.9%**	**47.6%**	**51.1%**	**40.4%**	**61.1%**
	7 Year Historical Volatility	39.2%	38.4%	N/A	58.8%	**38.8%**	**39.2%**	**43.8%**	**45.5%**	**49.0%**	**38.4%**	**58.8%**
	EWMA	46.3%	39.9%	79.8%	69.1%	**44.7%**	**57.7%**	**54.3%**	**58.8%**	**71.8%**	**39.9%**	**79.8%**
	Implied Volatility	44.8%	42.3%	86.0%	87.7%	**0.0%**	**42.3%**	**N/A**	**37.3%**	**65.4%**	**0.0%**	**87.7%**
	Company Used Volatility	39.9%	47.8%	49.5%	59.0%	**0.0%**	**39.9%**	**N/A**	**28.0%**	**48.7%**	**0.0%**	**59.0%**
	Selected Volatility	46.8%	40.7%	86.0%	60.2%	**45.2%**	**53.5%**	**53.9%**	**58.4%**	**66.7%**	**40.7%**	**86.0%**
	Volatility Size Adjustment	0.0%	0.0%	0.0%	0.0%							
	Size-Adjusted Volatility	46.8%	40.7%	86.0%	60.2%	**45.2%**	**53.5%**	**53.9%**	**58.4%**	**66.7%**	**40.7%**	**86.0%**
	Asset Volatility	18.0%	16.8%	86.0%	60.2%	**17.7%**	**39.1%**	**27.9%**	**45.3%**	**66.7%**	**16.8%**	**86.0%**

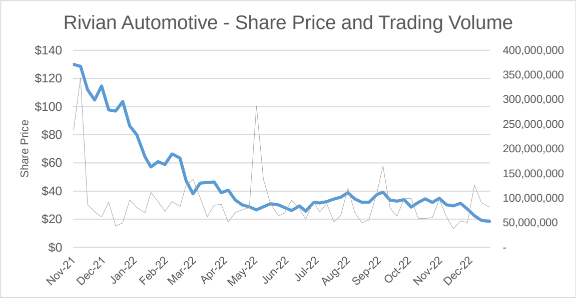
Volatility selected for analysis: 66%

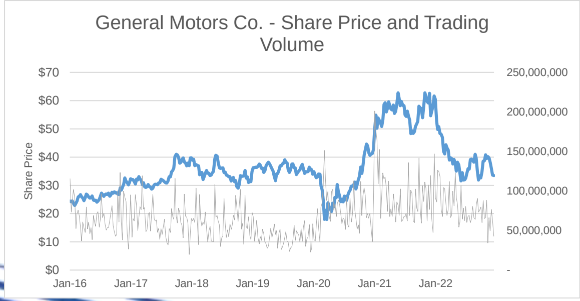
Notes: No size adjustment made due to high volatility of RIVN and TSLA; implied volatility selected for RIVN considering short trading history

Volatility Case Study (cont'd)

- Graphing the historical stock data is a good practice to identify data problems or abnormal trading activity







Volatility Analysis Case Study (cont'd)

Asset Volatility Calculation	F.US	GM.US	RIVN.US	TSLA.US
Market Capitalization	43,443	47,549	16,829	388,972
Net Debt	88,799	82,296	0	0
MVIC	132,242	129,845	16,829	388,972
Market Cap/MVIC	32.9%	36.6%	100.0%	100.0%
Equity Volatility	46.8%	40.7%	86.0%	60.2%
Asset Value	119,805	118,896	20,003	462,322
Equity Value	43,443	47,549	16,829	388,972
Exercise Price	88,799	82,296	0	0
Risk-free Rate	4.55%	4.55%	4.55%	4.55%
Term	3.0	3.0	3.0	3.0
D1	1.55130	1.87996	1.00000	1.00000
D2	1.23878	1.58914	(0.48992)	(0.04295)
N(D1)	0.93959	0.96994	0.84134	0.84134
N(D2)	0.89229	0.94399	0.31210	0.48287
Asset Volatility (Merton)	18.0%	16.8%	86.0%	60.2%
Asset Volatility (Static)	15.4%	14.9%	86.0%	60.2%

Volatility Analysis Case Study (cont'd)

- After re-levering the asset volatility, our concluded equity volatility for ABC Electric Vehicles is 78%:

Market Value of Invested Capital	1,500.0
Debt	350.0
Market Capitalization	1,218.8
Asset Volatility	66.0%
Equity Volatility	78.3%
Asset Value	1,500.0
Equity Value	1,218.8
Exercise Price	350.0
Risk-free Rate	4.55%
Term	3.0
D1	1.79865
D2	0.65550
N(D1)	0.96396
N(D2)	0.74393

Class-Specific Volatility Calculation

- For companies with preferred shares outstanding, the volatility of these preferred shares will generally be lower than the estimated overall equity volatility, while the volatility of the common shares will be higher
- We can estimate class-specific volatilities by extending the Merton Model:
  - Class Volatility = Equity Volatility x (Equity Value x Class N(d1)) / Class Value, where
  - Class N(d1) = Sum(Incremental N(d1) Value by Breakpoint x Class Allocation by Breakpoint
- The class-specific volatility for common shares can then be used to estimate DLOM for the common shares

Class-Specific Volatility Calculation Example

- Linero Co. recently raised $3.0m through issuing 500,000 series B preferred shares at $6.00. It previously had issued 1.0m common shares, 1.0m Series A preferred shares at $2.0, and 200,000 options with an exercise price of $3.50, giving a 100% equity value of $11.6m from OPM Backsolve analysis
- Linero's expected equity volatility is 57%, time to exit 5 years, and the risk-free rate 1.51%
- Class volatility for common shares is calculated as the sum of its delta spreads x (total equity value / share class value) x equity volatility, with the delta spreads calculated by subtracting the N(d1) values from one breakpoint to the next and security deltas by applying the relative value allocations

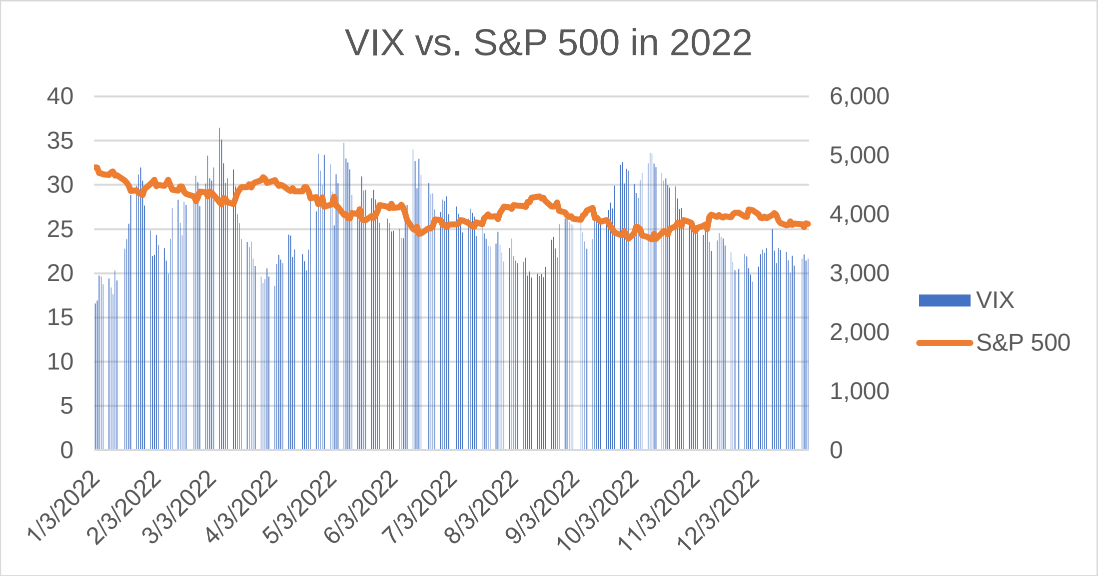
	Breakpoint 1	Breakpoint 2	Breakpoint 3	Breakpoint 4	Breakpoint 5	Sum	
N(d1)	1.0000	0.9130	0.8634	0.7927	0.6815		
Delta Spreads	0.0870	0.0496	0.0707	0.1112	0.6815		
Security Delta							Class Volatility
Common Shares	-	0.0496	0.0353	0.0505	0.2524	0.3879	67.30%
Series A Preferred Shares	0.0348	-	0.0353	0.0505	0.2524	0.3731	57.14%
Options	-	-	-	0.0101	0.0505	0.0606	83.07%
Series B Preferred Shares	0.0522	-	-	-	0.1262	0.1784	39.46%

Volatility Indices

- Volatility indices provide a quantitative measure of expected market volatility. These indices track the **implied volatility** of a specific indices and serve as a barometer of market sentiment and can help investors gauge market stability or anticipate potential market downturns
- Several indices of implied volatility are available
    - VIX – implied volatility of S&P 500
    - VXN – implied volatility of Nasdaq 100 (largest non-financial stocks)
    - RVX – implied volatility of Russell 2000 (small cap stocks)
    - VXD – implied volatility of Dow Jones Industrial Average
- Indications from volatility indices could be considered instead of using groups of guideline public companies for certain calculations, such as DLOM's

Volatility Indices - VIX

- The VIX index is considerably more volatile than the S&P 500:



Questions?

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Appendix

Merton Model

- The Merton Model is named after Robert C Merton, an American economist Professor at Massachusetts Institute of Technology (MIT). He developed the model in early 1970s as an extension of Black-Scholes-Merton option pricing model.
- The Black-Scholes-Merton model, developed by economists Fisher Black and Myron Scholes in 1973, is widely used to price options and derivatives. Merton recognized that the principles behind option pricing could also be applied to corporate debt and credit risk assessment.
- Merton's key insight was that a company's equity can be viewed as a call option on the value of its assets, with debtholders acting as a counterparties to this option. If the value of company's asset fall below its debt obligations, then the equity holders may choose to walk away or default leaving the debtholders with the remaining value.
- Based on this idea, Merton formulated a mathematical framework to analyze the relationship between company's asset value, its liabilities i.e., debt, and probability of default. He applied the option pricing technique from Black-Scholes model to calculate the expected default risk and the value of corporate debt.

Merton Model (cont'd)

Merton Model Formula:
E = V * N(d1) – D * exp(-Rf * T) * N(d2)
d1 = (ln(V/D) + (Rf + $\sigma 2 v$/2) * T) / σv * sqrt(T)
d2 = d1 - σv * sqrt(T)
Where:

- E = Equity Value
- V = Asset Value
- D = Debt Value
- N = Cumulative normal distribution
- Rf = Risk-free rate
- T = Future time period

If, V(t) < D(t) => E(t) < 0

- Equity holders may walk away
- Creditors claim are not fully covered
- Company is in default

Therefore, Probability of Default is:
P(Vt < Dt) = N(-d2)
N(-d2) is used in valuation of put option using Black-Scholes-Merton model.

Merton Model (cont’d)

- The Merton Model is based on several key assumptions and logic to assess the credit risk of the company:
    1) Asset Value: The model assumes that the value of a company's assets follows a geometric Brownian motion, which means that it fluctuates randomly over time. This assumption is consistent with the observation that asset prices in financial markets tend to exhibit stochastic behavior.
    2) Relationship between assets and liabilities: The Merton model considers the relationship between a company's assets and its liabilities (debt). It assumes that the company's debt holders have a claim on the company's assets and that the value of debt is dependent on the value of assets.
    3) Option-like characteristics: The equity of a company is viewed as a call option on the value of its assets. The option-like characteristics arise from the fact that equity holders have the right, but not the obligation, to claim the residual value of the company's assets after fulfilling the obligations to debt holders. If the value of assets is high, equity holders’ benefit; if the value of assets is low, equity holders may choose to default.
    4) Default and insolvency: According to the Merton model, default occurs when the value of a company's assets falls below its debt obligations. In this case, equity holders may choose to default, leaving the debt holders with the remaining value. The model calculates the probability of default based on the likelihood of asset values falling below the debt threshold.
    5) Calculation of credit risk metrics: By applying the option pricing techniques from the Black-Scholes model to corporate debt, the Merton model calculates credit risk metrics such as the probability of default, the expected loss given default, and the credit spread. These metrics provide insights into the creditworthiness of the company and can be used for risk assessment and pricing.
- The Merton Model makes simplifying assumptions and relies on certain inputs, such as volatility estimates and the risk-free interest rate. These assumptions and inputs can impact the accuracy and reliability of the model's outputs.

EWMA Volatility Calculation

- Exponentially Weighted Moving Average (EWMA) is a method in which weights are reduced exponentially for each period that goes back in time
- The result from the calculation is cumulative and all results contribute to the result, but the contribution factor declines as you move further back in time
- The EWMA formula is:
  - EWMA(t) = λ * (x(t) + (1- λ) * EWMA(t-a)
  - Where EWMA(t) = moving average at time t, λ = degree of mixing parameter value (decay factor) of 0 to 1, x(t) = value of signal x at time t, which is normally the squared, normal log of the price change
- A typical value for λ is 0.94, although 0.80 – 0.99 is a common range
- The total sum of the calculations represent the expected daily variance in stock
- EWMA assumes that data is normally distributed

GARCH Volatility Calculation

- The GARCH (general autoregressive conditional heteroeskedasticity) is a statistical model used in analyzing time-series data where the variance error is believed to be serially autocorrelated. GARCH models assume that the variance of the error term follows an autoregressive moving average process.
- GARCH models are used when the variance of the error term is not constant. That is, the error term is heteroskedastic. Heteroskedasticity describes the irregular pattern of variation of an error term, or variable, in a statistical model.
- A GARCH model uses values of the past squared observations and past variances to model the variance at time t.
- GARCH models allow for the modeling of this volatility clustering phenomenon by introducing a time-varying conditional variance component. The conditional variance is the squared value of the error term in a regression model, and it represents the volatility of the financial returns.

GARCH Volatility Calculation (cont'd)

Generalized Form of GARCH(m,n):

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{n} \beta_j \sigma_{t-j}^2$$

$$\varepsilon_t = r_t - \mu_t$$
$$\varepsilon_t = \sigma_t$$

Where:

- σ_t^2 is the conditional variance at time t
- ε_t is the standard residual (difference between observed value and the expected value)
- $\alpha_0, \alpha_1, \alpha_2, \ldots\ldots, \alpha_i \; and \; \beta_1, \beta_2, \ldots\ldots, \beta_j$ are the model parameters that need to be estimated
- Because conditional variances must be nonnegative, the coefficients in GARCH model are often constrained to non-negative.
- E(ε_t) = 0
- Var(ε_t) = $\frac{\alpha_0}{1 - \sum_{i=1}^{\max(m,n)} (\alpha_i + \beta_i)}$

Volatility Indices – VIX

- The VIX, also known as the “fear index”, is a widely recognized measure of market volatility and investor sentiment. Its primary purpose is to gauge the expected volatility of the US stock market over the next 30 days. The VIX provides insights into market participants expectations and perceptions of risk.
- The VIX was introduced by Chicago Board Options Exchange (CBOE) in 1993 as way to measure market volatility independently of stock price. It was designed to help investors and traders assess market risk, make informed trading decisions and manage portfolios effectively.
- The calculation of VIX is based on a complex methodology that utilizes options prices from S&P 500 index. It follows a concept of implied volatility, which represents the market’s expectation of future volatility derived from option prices.

Volatility Indices – VIX

- Calculation of VIX is based on a selected S&P 500 call and put options, using near-term and next-term options with sufficient liquidity and market participation to be useful
- For each individual options contract an implied volatility is calculated using Black-Scholes model
- The next step involves weighting the individual implied volatilities. Options with more time to expiration are given higher weightings, reflecting the greater influence they have on the VIX calculation. The weighting is determined based on a precise formula that takes into account the time remaining until expiration.
- The weighted implied volatilities are squared. This step emphasizes the impact of each individual option's implied volatility on the VIX calculation. Interpolation is used to fill in missing volatility values
- The squared, weighted implied volatilities are summed up to obtain the VIX calculation. The sum is multiplied by a constant factor to arrive at a percentage value that represents the expected annualized volatility of the S&P 500 index over the next 30 days.

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